DSC 140A - Discussion 02

Problem 1.

Let A be a *symmetric* $n \times n$ matrix, with entries:

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}.$$

Suppose $\vec{x} \in \mathbb{R}^n$.

a) Show that $\vec{x}^T A \vec{x} = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j a_{ij}$.

Solution:

$$\begin{aligned} \vec{x}^T A \vec{x} &= \vec{x}^T \sum_{j=1}^{n} x_j \begin{pmatrix} a_{1j} \\ \vdots \\ a_{nj} \end{pmatrix} \\ &= \sum_{i=1}^{n} x_i \sum_{j=1}^{n} x_j a_{ij} \\ &= \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j a_{ij} \end{aligned}$$

b) Show that

$$\frac{\partial}{\partial x_1} \left(\vec{x}^T A \vec{x} \right) = 2 \sum_{j=1}^{n} x_j a_{1j}$$

Solution:

$$\begin{aligned} \frac{\partial}{\partial x_1} \left(\vec{x}^T A \vec{x} \right) &= \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j a_{ij} \\ &= \sum_{j=1}^{n} x_1 x_j a_{1j} + \sum_{i=1}^{n} x_i x_1 a_{i1} \\ &= 2 \sum_{j=1}^{n} x_1 x_j a_{1j} \quad (\text{as } \; a_{1j} = a_{j1}) \end{aligned}$$